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FORMATION OF THE LOGICAL COMPONENT OF COGNITIVE UNIVERSAL LEARNING ACTIONS IN THE PROCESS OF TEACHING JUNIOR SCHOOLCHILDREN THE SOLUTION OF SIMPLE PROBLEMS

Abstract. According to the adopted second-generation FSES, universal learning actions are to be acquired by students on the basis of all academic subjects. It is possible to form logical universal learning actions at the lessons of mathematics while teaching problem solving. The analysis of the literature in methods has shown that practically no educational system has considered the concept “problem” from the conceptual point of view. From the conceptual point of view, the problem is a mathematical story about a quantitative change of the initial number. This story contains a question, to answer which one is to perform an arithmetic operation. The article addresses one of the approaches to the formation of logical learning actions via work on a simple problem. There are two debatable issues in the modern methodological literature. The first issue is about the role of problems in the course of primary school mathematics, the second is about the approaches to teaching problem solving. On the one hand, teaching problem solving is considered as the goal of education (the child must learn to solve problems), and on the other hand, the process of teaching problem solving is regarded as one of the means of mathematical, and specifically logical, and, in general, intellectual development of the child. This problem should be solved in the context of the thesis of L.S. Vygotskiy, from which it follows that teaching and development form a unity. In our opinion, the decisive role in this question is played by the methods of teaching problem solving, which either ensure or do not ensure the development of mathematical, and specifically logical, thinking.

Keywords: universal learning actions; logical component; simple problems; math problems; problem solution; cognitive activity; primary school; junior schoolchildren; methods of teaching mathematics at school; primary education in mathematics.

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The term "universal learning actions" (ULA) presupposes the ability of a subject to realize self-development and self-perfection via conscious and active acquisition of new social experience. In psychological context, the term may be defined as a complex of the pupil's actions ensuring their ability to independently acquire new knowledge and skills, including the organization of this process.

The following kinds of universal learning actions are singled out: personal, regulatory, cognitive and communicative.

The given paper dwells on the cognitive UAL, and specifically on one of its components – the logical one.

According to the FSES [12], cognitive UAL include the following actions.

- Can formulate problems and solve them.
- Can build up a logical sequence of suppositions.
- Can create oral and written utterances.
- Can structure the information obtained in the necessary form.
- Can choose the most suitable method of problem solving for the given situation.
- Can perform the operations of seriation and classification, can distinguish causative-consecutive relations.
- Can analyze the course and method of action performance.
- Can read reflectively, extracting the necessary and throwing away secondary information.
- The skills of analysis and synthesis are well-formed in the child.
- Can conduct choice and single out the necessary information.

In their turn, universal logical actions include the following positions:

– analysis of objects with the focus on distinguishing qualities (significant and non-significant);

– synthesis as a process of constructing a whole from its parts, and specifically in the course of independent addition of the missing components;

– choice of foundations and criteria for comparison, seriation and classification of objects;

– establishment of correspondence to the notion, recognition of objects;

– establishment of causative-consecutive connections, construction of a logical train of assumptions and proofs;

– detection of generic features and situationally relevant properties.

The UAL mastered by the children ensure acquisition of the key competences constituting the basis of the learning skills, and of the interdisciplinary notions.

According to the adopted second-generation FSES, universal learning actions are to be acquired by students on the basis of all academic subjects. It is possible to form logical universal learning actions at the lessons of mathematics while teaching problem solving.

The analysis of the literature in methods has shown that practically no educational system has considered the notion "problem" from the conceptual point of view.

From the conceptual point of view, the problem is a mathematical story about a quantitative change of the initial number. This story contains a question, to answer which one is to perform an arithmetic operation.

Taking into account what has been said above, we can formulate the content of the notion "problem":

– presence of at least one known number;

– presence of at least one unknown number;

– relation between the known and unknown numbers which allows calculating unknown numbers via arithmetic operations.

In the light of these requirements, all problems to be solved in primary school classes may be divided into three groups: elementary, simple and composite.

An elementary problem has the following significant features:

– one known number;

– one unknown number;

– relation between the known and unknown numbers which allows finding the unknown number via reasoning;

– is solved logically.

A simple problem has the following significant features:

– two known numbers;

– one unknown number;

– relation between the known and unknown numbers which allows finding the unknown number via arithmetic operation;

– is solved by one arithmetic operation.

A composite problem has the following significant features:

– two or more known numbers;

– more than one unknown number;
– relation between the known and unknown numbers which allows finding the unknown numbers via arithmetic operations;
– is solved by more than one arithmetic operation.

The group of simple problems is more interesting for research, because the ability to solve such problems makes it possible to understand the solution of both logical and composite problems.

There are two debatable issues in the modern methodological literature. The first issue is about the role of problems in the course of primary school mathematics, the second is about the approaches to teaching problem solving.

The first issue aroused discussion in connection with the fact that in the second half of the 20th century there was a change of paradigms in views upon teaching and development. There appeared two points of view on the role of problems in the course of primary school mathematics.

On the one hand, teaching problem solving is considered as the goal of education (the child must learn to solve problems), and on the other hand, the process of teaching problem solving is regarded as one of the means of mathematical, and specifically logical, and, in general, intellectual development of the child.

This problem should be solved in the context of the thesis of L.S. Vygotskiy, from which it follows that teaching and development form a unity. In our opinion, the decisive role in this question is played by the methods of teaching problem solving, which either ensure or do not ensure the development of mathematical, and specifically logical, thinking.

At present, it is possible to single out several approaches to teaching problem solving [1; 2; 4; 10; 13; 15].

In the 60s of the 20th century, E. M. Semenov suggested and tested the notional approach to the introduction of the concept “problem” [9]. The specificity of his method consists in the fact that it rests on a correct classification of simple problems. It means the following.

The following problems are given.

– There were three apples on the plate. Two more apples were put there. How many apples are there on the plate now?
– There were three apples on the plate. Mother gave one apple to her daughter. How many apples are left on the plate?
– Three rabbits were given two carrots each. How many carrots were given to the rabbits?
– There were two books on the first shelf, and five books – on the second one. How many books more were there on the second shelf than on the first one?

– There were six books on the first shelf, and two books – on the second one. How many times more were there books on the first shelf than on the second one?

The comparison of the problems shows that, in spite of the difference in the plot, it is possible to figure out common features:

– two known numbers;
– one unknown number;
– the numbers have nominations, identical or different;
– there are words that explain the relation between the known and unknown numbers of the problem.

Words in a problem are called key words if they help:

– to identify the group of problems,
– to determine the kind of problem.

It follows that key words serve as a basis for classification of all simple problems into groups, and their relation to the known and unknown numbers of the problem – as a basis for classification of the problems within the group into concrete kinds.

Taking all this into account, we can subdivide all simple problems into 5 groups.

Group 1 – problems in which there is the key word "vsego" or its synonyms.

Group 2 – problems in which there are the key words "vsego", "ostalos'" or their synonyms.

Group 3 – problems in which there are the key words "po", "vsego".

Group 4 – problems in which there are the key words "na… bol'she (men'she), chem".

Group 5 – problems in which there are the key words "v… bol'she (men'she), chem".

This classification is shown in detail in Figure 1.

Let us denote the known numbers in the problem as A or B, and the unknown numbers – as X, then "KW … → A" should be read as "the key word … refers to the known number", and "KW … → X" should be read as "the key word … refers to the unknown number".

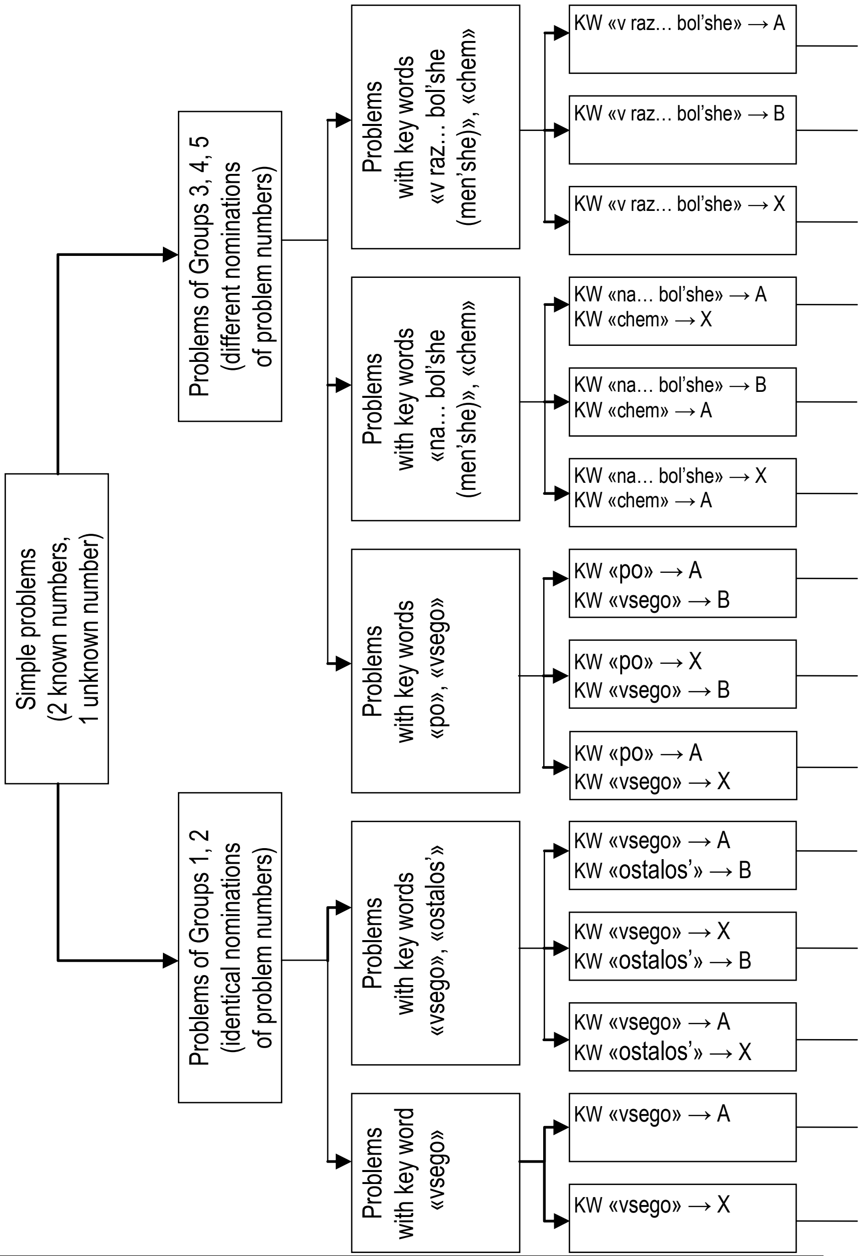
We will demonstrate the use of such logical actions as analysis, comparison and establishing correspondence to the notion on the example of the "problem on addition".

It is necessary to pass several stages in the course of the work.

1. To introduce the idea about the mathematical meaning of the key words "vsego", "stalo", because these words are often used by children in their everyday but not mathematical sense.

The following exercises can facilitate it.

– Take 2 rods in the right hand, and 3 rods in the left hand. Put them before you on the table. How many rods are there before each of you?


Simple problems (2 known numbers, 1 unknown number)
Problems of Groups 1, 2 (identical nominations of problem numbers)
Problems of Groups 3, 4, 5 (different nominations of problem numbers)
Problems with key word «vsego»
Problems with key words «vsego», «ostalos'»
Problems with key words «po», «vsego»
Problems with key words «na… bol'she (men'she)», «chem»
Problems with key words «v raz… bol'she (men'she)», «chem»
KW «vsego» → X
KW «vsego» → A
KW «vsego» → A KW «ostalos'» → X
KW «vsego» → X KW «ostalos'» → B
KW «vsego» → A KW «ostalos'» → B
KW «po» → A KW «vsego» → X
KW «po» → X KW «vsego» → B
KW «po» → A KW «vsego» → B
KW «na… bol'she» → X KW «chem» → A
KW «na… bol'she» → B KW «chem» → A
KW «na… bol'she» → A KW «chem» → X
KW «v raz… bol'she» → X
KW «v raz… bol'she» → B
KW «v raz… bol'she» → A

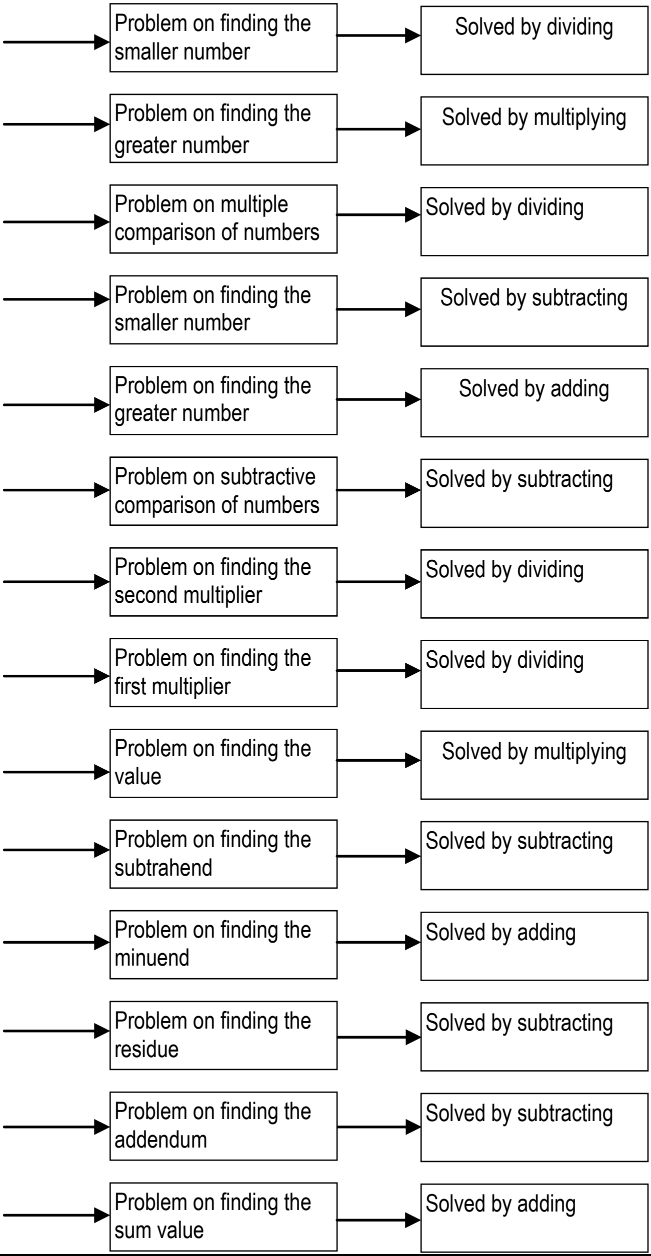


Figure 1. Classification of simple problems according to E.M. Semenov.

It is necessary to do 2-3 similar exercises with concrete objects (notebooks, pencils, books) attracting attention to the words “vsego”, “vmeste”, “stalo”, “v dvukh”. In this case, the pupil performs operations with mobile objects practically; the teacher stresses the key words vocally. Comparing the situations, the child arrives at the conclusion that it is necessary to unite the sets of objects in order to find the result.

After this, we can pass on to the problems in which it is impossible to perform union of the sets practically, it can be done only mentally, but there are also the words “vsego”, “stalo”.

Problem: “There are 5 TVs on one shelf, and 1 TV on the other shelf. How many TVs are there on two shelves?”

Conclusion: if it is necessary to find “skol’ko vsego” (*how many*), we must add the numbers given in the problem.

The key word that helps to choose the right operation is introduced in this way.

2. To figure out the relation between the key word and the unknown number of the problem. To do this, let us make up a problem opposite to the given one. Keeping in mind that the notion is introduced in grade 2, we must give the pupils a detailed instruction.

We will start work with the problem in which it is necessary to answer the question “Skol’ko vsego?” (*How many?*).

Kolya had 5 pencils. Anton gave him one more pencil. How many pencils did Kolya have now?

— Who has guessed what operation is used to solve the problem?

— The problem is solved with the help of addition.

— Why?

— The problem question is “How many pencils did Kolya have now?”

— Now, let the number of pencils Kolya had be unknown, and the number of pencils he will have is known; we’ll get the following problem: “After Anton had given Kolya 1 pencil, Kolya had 6 pencils. How many pencils did Kolya have previously?

— The word “stalo” in this problem refers to the known number 6.

— Has the operation used to solve the problem changed?

— Yes! We shall solve the task with the help of subtraction.

— Then, if the words “stalo”, “vsego” refer to the unknown number, the problem should always be solved using addition.

At this stage, having solved 2-3 problems on finding the sum value, we compare them and come to the conclusion that the problems in which the word “vsego” refers to the unknown number are called problems on finding the sum value.

3. Single out the known and the unknown numbers in the problem.

Special exercises on finding known and unknown numbers in the problem should be done at mathematics lessons. Here is an example.

Problem: “Sasha picked first 2 apples, and then 1 apple more. How many apples did Sasha pick on the whole?”

— Put the character corresponding to the number of the apples picked initially on the demonstration board.

— Put the character corresponding to the number of the apples picked later on the demonstration board.

— So these numbers are known: 2, 1.

— Put the character corresponding to the number of all apples picked by Sasha. Can you do so?

— If we don’t calculate it, we can’t.

— Why?

— We don’ know how many apples there were.

— Then this number is unknown. There two known numbers and one unknown number in the problem. In order to find it, it is necessary to solve the problem.

It is advisable to give problems with insufficient conditions.

Problem: “Petya cut out some stars, and then two stars more. How many stars has Petya cut out on the whole?”

— Can the problem be solved?

— No, as there is only one known number in it.

— Change the problem so that it could be solved by adding.

It might be useful to give problems with excessive information.

Problem: “There were two notebooks on the table. One more notebook was put. There were now three notebooks on the table. How many notebooks were there on the table finally?”

— What is unusual in the problem?

— All numbers are known there.

— Change the problem so that it could be solved by adding.

4. Define the role of nomination.

In order to show children that nomination also plays an important role in the problems on finding the sum value, it is useful to give problems of the following kind.

Problem: “There are 2 books on one shelf, and 5 notebooks on the other shelf. How many books are there on two shelves?”

— Can the problem be solved by adding?

— No, there are different names – books and notebooks.

— Change the problem so that it could be solved by adding.

The problem can be also changed so that it would contain a general word, and in this case the nominations will be considered identical.

Conclusion: If there are two known and one unknown numbers in the problem, the numbers have identical nominations, and there is the key word “vsego” which refers to the unknown number, it is a problem on finding the sum value. Such problems are solved by the operation of addition.

Hence, the analysis of a simple problem at the first stage of teaching will take the following form:

1. Name one known number
2. What does it mean?
3. Name the second known number.
4. What does it mean?
5. What is the main word in the problem?
6. What kind does the problem belong to?
7. What operation is this problem solved by?

At the next stage, we introduce problems on finding the addendum, in which schoolchildren get better understanding of the role of the relation of the key word to the known and unknown numbers of the problem. From the point of view of teaching methods, this kind of problem solving should begin with stage 2.

Similar work is carried out while introducing other kinds of problems as well. At the final stage, problem analysis is performed according to the following plan:

1. What kind of problem is it by composition?
2. Why do you think so?
3. What kind of problem is it?
4. Why do you think so?
5. What operation are these problems solved by?

Such approach to the introduction of the notion of “simple problem” automatically acquaints the child with the rule of establishing correspondence to the notion: if an object has all significant features of the given notion, it refers to this notion; if at least one feature of the given notion is absent it will be a different notion.

The work on other kinds of problems is continued and reinforced while solving composite problems.

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