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DEVELOPMENT OF MATHEMATICAL SPEECH IN PRIMARY SCHOOL

Abstract. The culture of mathematical speech is an integral part of the human culture. If well developed, it provides conscious acquisition of the mathematics course content by primary school pupils, formation of communicative learning actions, and achievement of subject, metasubject and personal learning outcomes. Analysis of teaching practice demonstrates a low level of development of mathematical speech of junior schoolchildren.

Deficiencies in the development of mathematical speech of primary school pupils are the result of a lack of theoretical and methodological elaboration of many aspects of this issue. It therefore seems a promising task to search for means of improving the formation of mathematical speech skills and habits.

In modern literature, there are several approaches to the study of mathematical language: the semantic approach and the syntactic one. The combination of these approaches to the construction and study of mathematical speech means that the grammatical rules of literary language, constructions of mathematical and logical terms should get a semantic interpretation also in those cases when they are formulated as syntactical.

Semantic and syntactic relations should be considered in conjunction with educational activity. The theory of speech activity can serve as a psychological basis of mastering mathematical speech.

Semantic skills are based on the action of semanticization of language units, consisting in the correlation of the sign with its meaning in thinking. Syntactic skills are based on the rules of construction and transformation of language units.

Skills of semiotic modeling and interpretation of formal mathematical expressions based on the analysis of the structure of mathematical language have been revealed in addition to the above mentioned ones. Teaching mathematics to junior schoolchildren uses both natural and special language — the language of Mathematics. It is at the primary school that the pupils come across the first artificial language of mathematics.

Work on the development of mathematical speech can be carried out in two ways: through imitating the teacher and through purposeful training. If the teacher would deliberately pay attention to the development of the pupils' conceptual apparatus of mathematics, speech would be a means of mental activity development and a basis for the formation of communicative learning actions.

Keywords: speech, speech activity, mathematical speech, techniques of mathematical speech development, primary school.

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Questions of children's speech development take a special place among the problems of general development of junior schoolchildren. As mathematical objects are part and parcel of the existing reality, so the mathematical speech culture is an obligatory constituent of the general culture of man. Mathematics, the same as other subject fields, makes certain contribution to the development of speech of a schoolchild. Well developed speech facilitates conscious acquisition of the content of the course of mathematics by primary school pupils, formation of communicative learning actions and achievement of subject, metasubject and personal learning outcomes.

Analysis of teaching practice reveals a low level of development of mathematical speech of junior schoolchildren. This is manifested in the fact that pupils have problems in a number of learning situations, when, for example, it is necessary to:

– substantiate the solution or point of view;

– understand and, consequently, do the whole task independently;

– formulate a learning problem, supposition or hypothesis;

– make a generalization, conclusion, etc.

Deficiencies in the development of mathematical speech of primary school pupils to a great extent may be attributed to inadequate theoretical and practice-oriented study of the methods and techniques of work in this direction. The search of means of improvement of the process of formation of mathematical speech skills and habits appears to be an important line of development.

Analysis of the literature on the problem under study reveals the need to specify the following moments. Language, including mathematical one, is defined as a system of verbal signs which is relatively independent of the individual and serves the purposes of communication, formation and formulation of thoughts, reflection and translation of the structure of society and historical experience. Speech is language in action; it is always a concrete process of using linguistic signs; it is a specific human way of formation of thoughts with the help of linguistic means. Mathematical terms, symbols, schemes, graphs, diagrams, etc. refer to such means. Thus, mathematical speech is the sum total of all linguistic means with the help of which one can express mathematical content.

Modern literature describes several approaches to the study of mathematical language: semantic and syntactical [1].

Semantics studies linguistic signs and expressions from the point of view of their meaning and defines the meaning of each mathematical sign. Syntax studies the rules of formation of linguistic units in relation to their meaning. In mathematical speech,

syntax regulates the rules of usage of mathematical signs in expressions, equations, inequations and other sentences of mathematical language.

The combination of these approaches to the construction and study of mathematical speech means that the grammatical rules of literary language and constructions of mathematical and logical terms should get a semantic interpretation, including those cases when they are formulated as syntactical.

Semantic and syntactical relations in mathematical speech should be regarded together with the activity for their acquisition. The activity theory worked out in Russian psychology by S. L. Rubinshteyn [11], A. N. Leont'ev [8], V. V. Davydov [4], D. B. El'konin [15], N. A. Menchinskaya [9] and other scholars may be taken as a psychological basis of mathematical speech acquisition. According to this theory, any activity is made up of actions, and actions consist of operations. An ability to perform an action is called a skill, and an ability to perform an action automatically is known as a habit. Accordingly, a speech habit is a speech operation conducted automatically; and a speech skill is an ability to use obtained knowledge and habits in various communicative situations.

In teaching mathematics to junior pupils both natural colloquial language and special mathematical language are used. Learning mathematical language and acquaintance with its components makes up an inseparable part of primary mathematics education. It is at the primary school that children get to know the man-made language of mathematics for the first time. That is why special attention should be paid to the work with its signs.

On the basis of analysis of the mathematical language structure, peculiarities of semiotic activity in scientific cognition, logico-cognitive processes of use of mathematical language in various situations, V.A. Drozd [5] singles out the following skills which provide acquisition of mathematical speech: semantic skills, syntactical skills, skills of semiotic modeling and interpretation of formal mathematical expressions.

Semantic skills are based on the process of semanticization of linguistic units consisting in establishing correlation between the sign and its meaning in thinking. Semanticization skills include all actions characterizing the process of notions acquisition:

– recognition of mathematical objects by their terms or symbols among other objects or images, identification of relevant properties and reproduction of notions, evaluation of the correspondence of verbal or symbolic expression to an object-material or material situation;

– matching a mathematical object with a notion, definition of ties between them;

– reproduction of object situations typical of mathematical reality in a verbal-symbolic form, performing mental operations in mathematical terms and symbols.

Syntactical skills are based on the rules of construction and transformation of linguistic units. The structure of symbolic mathematical expressions is studied on the basis of their

comparison with sentences of the natural language and is expressed in the skills of:

– reading and writing mathematical expressions;

– transformation of expressions in accordance with the rules established in mathematics.

Semiotic modeling depends on semantic and syntactical skills. The operational aspect of modeling skills includes actions in revealing the object of the task, connections between the objects and connections between the connections.

The basic components of the operational aspect of the skill to interpret formal mathematical expressions include:

– delimitation of the object field taking into account correlation between objects and elementary symbols;

– revealing peculiar features of the syntactical structure in question;

– definition of connections between objects satisfying the syntactical structure in question.

Thus, analysis of skills which allow developing mathematical speech in primary school shows that most attention should be concentrated on:

– the junior schoolchildren's understanding the meaning of mathematical notions;

– the formation of skills to establish semantic relations between the notions, terms and symbols, translate life situations into the language of mathematics and represent these situations in various mathematical models.

It is also rather useful to perform the opposite operation – to interpret the information provided in the mathematical language by means of a natural language.

It should be noted that at junior school age speech development of children is effected in two ways: through imitation of speech of the surrounding people and, first of all, their teacher, and by specially organized teaching. Development of coherent speech through imitation is especially effective at the initial stage of subject learning as junior schoolchildren still possess a unique ability to "absorb" speech patterns and at the same time are already prepared to mastering various structures of mathematical speech.

In this connection it is necessary to note that the speech of a primary school teacher is not only the main instrument of professional activity and not only a form of teaching the subject but also a technical means of teaching. It is at primary school that the teacher should give the fundamentals of linguistic knowledge about the language as a means of communication and cognition, tell the pupils about the laws of its functioning (maybe in a simple understandable form) and formulate the basic requirements to speech in general and mathematical speech in particular.

Purposeful teaching presupposes provision of the following conditions:

– creation of positive motivation to learning mathematical speech;

– use of a system of special exercises initiating the process of formation and development of mathematical speech;

– organization of such education by means of which the school-

child is constantly involved in active speech in the process of independent search of new information and usage of mathematical speech.

Creation of positive motivation to learning mathematical speech forms one of the initial stages of the process under study. For this end, the teacher provides the pupils with elementary information about the following: what we need usual colloquial and mathematical speech for, what is an utterance, what kinds of utterances are there, how we produce an utterance, conclusion, statement. And especially important are the tasks that develop critical evaluation of one's own and somebody else's speech and the feeling of communicative relevance. Further work consists in teaching:

– oral reproduction of a learning task, plan of its fulfillment, explanation of the process and result of the task solution;

– making inductive and deductive utterances in the process of substantiation of one's utterances;

– operating logical links "no", "and", "or" and logical words "some", "every", "any".

For organization of active speech of the pupils it would be useful to work out a system of special exercises in the process of which the teacher should:

– help the children realize their speech and on this basis teach them how to master the skills of talking and coming to an agreement;

– create the situation of speech interaction in the classroom modeling real communication (pair work, group work);

– make the pupils express their attitude to a certain fact, event or phenomenon;

– make the pupils use the learnt speech material;

– attract the attention of the pupils to the content of utterances;

– presuppose the formation of various kinds of coherent speech: description, reasoning, argument, foundation, explanation, planning and generalization;

– carry out systematic work towards acquisition of norms of mathematical speech presupposing realization of the following aspects:

- work with vocabulary (lexical level);
- work with phrases and sentences (syntactical level);
- work with coherent speech — logical construction of utterances (text level).

Let us now describe some concrete techniques aimed at the development of mathematical speech.

The "language" of textbooks is one of the necessary conditions of development of correct mathematical speech of the pupils. It must surely present a model of logical perfection because the pupils makes their first steps in mastering mathematical terminological vocabulary on the example of learning texts by imitating them. Analysis of mathematics textbooks for primary school shows that many of them are characterized by linguistic and logical negligence. It is important to note that the new generation textbooks often overcome this drawback. The pupils' thinking is expressed in speech; it means that the

teacher should organize systematic work with the aim of the pupil's mastering linguistic means which would influence the development of thinking.

A most widely spread and effective technique of such work may consist in making pupils give full and correct explanations of the performed actions. For example, answers at the blackboard, "from seat", "after classmate" are widely used by teachers while working on the algorithm of reasoning and learning task planning. Answers should be corrected with the help of additional questions, involving pupils in the process of correction, addition and specifying periphrasis of reasoning while solving a problem task.

Much effort is spent by the teacher on developing imagery, expressiveness and emotional charge of the pupils' speech. But perfection of speech in terms of its logic, sequence and precision is not always in the focus of the teacher's attention. This can be partly explained by the absence of the corresponding tasks in the textbooks and inadequate elaboration of these questions in guidelines for methods of teaching mathematics.

While compiling tasks for the development of mathematical speech it is important to formulate a concrete aim for each speech exercise. The tasks should be versatile and age-oriented. Now let us give some examples.

Task 1. Think of as many explanations to the phrase "the sum value" as possible.

Possible answers:

– summation result;

– a number which we get in the result of summation of two or more numbers;

– a number larger than any of the summands or equal to one of them if the other summand is equal to zero;

– a number from which one of the summands may be subtracted to get the other summand;

– a number which does not change if we change the order of the summands, etc.

Task 2. Constructing mathematical sentences. Give the children words which they are to use in a sentence or the wording of a known rule.

For example, make up a definition using the words: "expressions", "equality", "connected", "two", "by the sign", "this".

Task 3. Making up texts of problems according any possible model: scheme, drawing, expression, brief notation, etc.

For example, we can ask to make up a problem according to a drawing.

Task 4. Making up mathematical problems according to given characteristics. Explanation of the line of reasoning and proving its truth should be compulsory.

For example, given numbers 16, 4, 20. Task: make up a one step problem; a two steps problem; a problem with the question "How many?"; using the given numbers make up three true equations; a complex equation, etc.

Task 5. Read the words and put the stress: "kilometer, millimeter, calculate, nomenclature, etc."

Task 6. Explain the meaning of the mathematical terms: "expression", "calculation exercise", "inequality", "equality", "minuend", "subtrahend",

“complex problem”.

Task 7. Correct spelling mistakes in the mathematical terms: “exprassion”, “culculation exercise”, “unequality”, “equolity”, “mimuend”.

Task 8. Insert suitable words to make a correct utterance: “The sum value is a number which we get in the result of … of two or more numbers “; “To add a sum to a number we can …”

Task 9. Correct the style in the following explanations:

A. Explaining calculations in the expression (5 + 4) Kolya said as follows: “While adding to number 4 of number 5 we get 9”. What speech errors did Kolya make?”

B. Having completed an operation (18 + 2 = 20) Natasha said: “I got 20, I’ve made a correct calculation”. Could we consider her answer correct and full?

Special place is occupied in mathematical education by deductive and inductive utterances. One of the main tasks of the teacher is to teach pupils make up and use deductive and inductive utterances correctly; and the skill of producing such utterances is a sign of conscious and deep understanding of mathematical content. What is more, children’s ability to make up deductive and inductive utterances correctly is an inseparable part of the logical constituent of mathematical education.

The terms “deduction” and “induction” could be used in several meanings: method of argumentation, method of textbook material presentation, teaching method and a form of reasoning.

inductio(n-), meaning *inducement, coercion*).– inference of a generalized conclusion from particular instances;

– research method according to which separate objects (circumstances) are studied in order to invest a multitude of objects (phenomena). The qualities of the multitude are defined through the qualities of separate objects;

– form of material presentation in a work of literature or talk that moves from specific instances to a general conclusion.Methods and techniques of teaching junior pupils at the stage of acquisition of new knowledge in the majority of cases are connected with inductive reasoning. The translation of the term itself suggests didactic possibilities of this method: conclusions obtained in this way are based on observation, analysis, comparison, finding common features and their further generalization.

We observe in this case a close relationship between the methods of teaching and cognition, and the method of partial induction in particular. The essence of this cognitive method consists in the fact that observing various particular cases we notice a certain regularity which makes it possible to make a general conclusion. But it is necessary to keep in mind that usually it is impossible to embrace all particular cases; that is why reasoning based on partial induction cannot be regarded as truly scientific argumentation.

From the point of view of teaching methods, the method of partial induction has a number of strong points: development of logical form

of thinking (analysis, synthesis, comparison, and generalization), activization of cognitive activity of pupils, joy of discovery, acquaintance with a cognitive method used in science. While teaching inductive utterances the teacher should offer exercises aimed at the development of observation which is closely connected with the techniques of analysis, synthesis, comparison and generalization.

What is similar between the expressions and what do they differ in?

1)3 + 5 3 + 6

2)8 — 3 8 — 4

1. Compare the values of these expressions.

2. What conclusion can be made after their observation?

3. Make up pairs of similar expressions and check if your conclusion about them is true.

Pupils of second and third grades should be given tasks demanding independent discovery of connections and dependencies, and formulation of generalizations. The following task may be the most typical one: Compare the expressions. Calculate their values. Compare the obtained results. Figure out their common feature(s) and formulate the rule".

First grade pupils should be given a detailed plan of actions leading to conclusions and generalizations.

Task. Given expressions:

0 + 1 1 + 2

2 + 3 3 + 4

4 + 5 5 + 6

6 + 7

1. Compare the numbers in the expressions.

2. Think of how they can be named in relation to each other.

3. Calculate the values of the expressions.

4. Think of how the values of these expressions can be named by one word.

5. Make a conclusion.

Pupils may give different answers depending on the group they learn in:

– "The sum value of two sequential numbers is an uneven number";

– "The sum value of an even number and an uneven one is an uneven number".

In the process of teaching inductive reasoning it would be useful:

– to urge pupils to search for new examples corroborating the conclusion;

– to teach them to compare the conclusion with the facts on the basis of which it was made;

– to search for such facts that could disprove the conclusion.

In such cases we could also use the technique of intentionally making the pupil face the situation in which the obtained conclusion turns out to be wrong.

To ensure the succession between primary and junior secondary school, as well as between different subject fields it is necessary to teach deductive reasoning beginning with primary school.

As we have already stated primary school children get knowledge about properties, regularities and connections through induction and according to the results of measurement,

calculation, observation and comparison formulate a conclusion.

The use of the formulated conclusion should be effected by means of deductive reasoning which fosters rigorous, clear and laconic thinking.

An example of deductive reasoning. "We know that if we add 1 to any number we shall get the nearest following number. We have to add 2 and 1 to get 3, because 3 is the nearest following number of 2".

Task 1. Compare the numbers 5 and 8.The course of deductive reasoning: "If a number is used before another number in counting, this number is smaller than the other one. 5 comes before 8; it means that 5 is smaller than 8".

Task 2. Solve the problem and comment on the choice of operation.

Problem. Kolya has 6 stamps and Petya has 2 stamps. How many more stamps does Kolya have than Petya?The course of deductive reasoning: "In order to find out how much one number is bigger than another one we have to subtract the smaller number from the bigger one. In the problem we have to learn how many more stamps Kolya has than Petya? So we have to subtract the number of stamps Petya has from the number of stamps Kolya has".

In order to bring children closer to producing deductive utterances, it may be useful to introduce playing with fairy tale numbers [10].

$a \times 1$; $a \div 1$; $a + 0$; $a - 0$" we can do the following: first ask the pupils to calculate the values of the expressions $(5 \div 1)$; (3×1); $(6 \div 6)$; $(7 + 0)$; $(8 - 0)$ and comment on the obtained results.

(The children may give the following explanation: $5 \div 1 = 5$, because $5 \times 1 = 5$).After that we can suggest playing a fairy tale school game where all numbers are not like the numbers we use in our math, and only the numbers 1 and 0 have the same notation.

— "Imagine you are in a fairy tale school. Could you calculate the values of the following expressions in this case?

$y \div 1$

$w \div w$

$z - 0$"

Introduction of unknown signs makes children use deductive reasoning: "Division of any number by 1 gives the same number, so in the answer we shall write the same sign as was used to denote the first number".

We believe that the proposed approach and the techniques of developing mathematical speech described above do not only enlarge the vocabulary of mathematical terms but also foster interest to the science itself – to mathematics.

Quite often pedagogues associate the work on speech development only with learning Russian and reading Russian literature whereas any subject can make its own contribution to this process. What is still more, if the teacher would purposefully pay attention to the children's acquisition of the terminological apparatus of the learnt discipline we can hope that the task defined in the standard of general primary education (second generation) will be fulfilled, and speech would become a means of develop-

ment of intellectual activity and a basis for the formation of communicative learning actions.

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