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TEACHING PUPILS TO USE THE GENERAL METHOD OF SOLVING PROBLEMS

Abstract. This paper discusses the general methods of work with the learning problem as a basis for the formation of universal learning actions on the example of mathematics problem solving in primary school.

Keywords: universal learning actions, learning problem, general method of solving problems, stages of work on a mathematics problem, primary school.

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Formation of universal learning actions giving the schoolchildren a chance to acquire the skills of leaning, self-development and self-perfection is a most important task of the modern education system.

The educational standard of the second generation stresses that the skill to solve problems and to complete tasks is one of the main cognitive universal actions. The notion "problem" has several synonyms (task, purpose, question, etc.) and is widely used in various fields of science and practical activity (pedagogical problem, cognitive problem, technical problem, etc.).

In spite of such wide usage, the process of solving a problem in any field has a uniform structure: entering a situation demanding its detailed analysis, modeling, planning, realization of the plan, checking the correspondence of the result to the predetermined purpose.

For example, while teaching various subjects we use problems which are typically called learning problems. Solution of learning problems functions as an educational method. They are used to form meta-subject and subject knowledge, skills and habits. The skill to set and solve learning problems is one of the parameters of the level of pupils' development and opens up avenues for acquisition of new knowledge.

At the same time there are academic subjects – mathematics, physics and chemistry – in which pupils

solve the so called word problems. Solution of word problems at lessons in these disciplines is viewed upon as a subject of special investigation. So problem solving may be regarded both in a broad and narrow sense.

If we treat teaching word problem solving as teaching solving any problems in the broad sense of the word, then the skills formed in this process can be used for the solution of any problem (learning or practical) and in any sphere of thinking or practical activity. We called the skills formed in this case and allowing to solve any problem (learning or practical) general skills; the method of teaching such skills is referred to as the general method of problem solving.

The general method of problem solving at primary school should be an object of special training with a step-by-step work on each of its components. The method is based on formation of logical operations – ability to analyze the object, compare, find general and specific features, classify, build series, logical multiplication, and draw analogies. Acquisition of this method will enable pupils to analyze and solve various types of problems in the subject and use this skill for solution of problems in any other sphere of activity. Thus, due to its systemic character the given universal learning action may be considered as a model one for the system of cognitive actions [8].

The general method of problem solving formed on the basis of mathematics may be transferred as a general structure to any academic subject. In reference to the disciplines of the science cycle the content of the method needs few changes – the differences will be connected with the specific subject language of description of problem elements, their structure and kinds of symbolic representation of relations between them. The influence of the specificity of the discipline on the acquisition of the given universal learning action is reflected also in differences of semantic analysis of the text of the problem. For example, while solving mathematical problems it is necessary to abstract from the concrete situation described in the text and dwell on the structure of relations which connect the elements of the text. While solving the humanities problems, the concrete situation is not analyzed, as a rule, with the purpose of abstracting from its peculiarities but, on the contrary, is studied with the aim of revealing specific features for subsequent generalization of the obtained information [8].

Thus, the modern educational standard orients educational systems, practical teachers and the methods of teaching mathematics towards the formation of the general method of problem solving.

As we have already specified, teaching the general method of problem solving presupposes concentration of effort not on the process of getting the answer but on the formation of skills making pupils capable of solving any problem. Each of these skills should become an object of special training.

The general method of problem solving presupposes the knowledge of solution stages, solution methods and

means, types of problems, reasons for choosing arithmetic operations for calculation and knowledge of the subject material: rules, formulae, logical techniques and operations.

The modern literature in methods [6; 11; 13; 14; 15] contains several points of view on the question of stages of problem solving. Comparison of different approaches to staging makes it possible to state that differences exist only in terminology; the sequence and content of activity is subject to little variation. The main differences are observed in the techniques of teaching a certain general action constituting the technique.

The first stage of work on the problem and the initial component of the general method of problem solving consists in problem text analysis. It targets perception and understanding of the text of the problem.

Three kinds of problem text analysis are distinguished in mathematics: semantic, logical and mathematical [10].

The aim of these kinds of analysis is to guarantee understanding of the problem text.

Semantic analysis presupposes the following:

– distinction and understanding separate words, terms and notions of both everyday and mathematical character;

– realization of grammar constructions ("both…and", "as soon as …", etc.);

– defining quantitative characteristics of an object;

– presentation of the situation described in the problem by rewording or simplifying it including only the most essential information for its solution;

– specification of the essence of the problem, the object and the value to be defined (price, volume, square, number, etc.).

Logical analysis presupposes the ability to replace the terms characterizing notions (processes, criteria) with their definitions; ability to make conclusions on the basis of the data given in the problem.

Mathematical analysis includes analysis of the problem situation and task. Analysis of the problem situation is aimed at:

– naming objects (things or processes);

– considering the relationships between objects in terms of the whole and its parts;

– considering the number of objects and their parts or values characterizing each object;

– analysis of the characteristic features of the values (homogeneous or heterogeneous), numerical expressions of values (known and unknown);

– data changes analysis: whether they change (reference to the logical order of all changes) or do not change;

– revealing the relationships between known and unknown values;

– task analysis – specifying unknown quantitative characteristics of object values.

At this stage, the first moment –

first reading of the problem situation – is the most important one. This moment is underestimated in school practice. Quite often the child has even not enough time to read the text, to say nothing of its understanding, when the teacher call him to go to the blackboard to solve the problem. It is necessary to note that hasty transition to the transformation of information almost on the point of its reception without preliminary analysis affects the cognitive process. But attentive preliminary reading of the text, clear realization of the problem situation by the pupil allows making many useful conclusions and suppositions about the approaches to its solution.

Children are taught at this stage to extract text information relevant for the solution of the problem. It should be decided if this information is enough to solve the problem; unnecessary information should be dropped. Information reality is figured out if it is demanded by the plot of the problem. The problem text is transformed (either according to a given scheme, or in order to make the text easier for comprehension) leaving only mathematically relevant information.

Practical teachers worked out special techniques of working over mathematical problems at this stage. Usually, two main actions – problem situation reading and problem text oral reproduction – are formed at this stage.

The following techniques are used during oral reproduction of the problem text.

1. Abstracting number from the plot of the problem.

2. Reproduction of the problem by logical parts. This technique is used either at the initial stage of work or wile reproducing the text of a problem with unfamiliar plot.

3. Reproduction according to the structural parts of the problem.

4. Reproduction of the whole text of the problem.

Mathematical, logical and semantic analyses are carried out in accordance with the peculiarities of the problem using the following techniques:

– transformation of the problem text which presupposes dropping out the part which is not significant for the solution result or addition of the missing data;

– change of word or sentence order; replacement of some words by synonyms; substitution of a descriptive phrase by a term or vice versa;

– addition of text explanations; specification of the measure units of values, etc.

The text model of a problem often includes irrelevant information. In order to be able to work with essential semantic units the problem text is translated into the language of graphical models, i.e. the text is presented with the help of non-verbal means – models of various kinds: drawings, schemes, charts, tables, symbolic drawings, etc.

Translation of the text into the mathematical language with the help of non-verbal means constitutes the second stage of work over the problem. Realization of this stage (of the second component) presupposes the

choice of signemic-symbolic means for building a graphical model adequate to the mathematical content of the problem. The problem model built in accordance with certain rules is the problem's analogue in which the structure of relationships and connections between objects or values described in the problem are more clearly expressed. Transformation of a problem into a graphical model allows seeing textual properties and relationships which are hardly seen while reading.

After the text has been presented in the form of a graphical model it is necessary to pass on to the analysis of relationships and connections between the known values, as well as between known and unknown values. A detailed analysis of these relationships is carried out for this purpose. The result of this analysis allows one to build up a plan for the problem solution. That is why it would be logical to call this stage a stage of searching for a plan for the problem solution.

Methodological literature distinguishes direct analysis (synthesis), backward analysis and mixed (analytical-synthetic) method. Each of these kinds of analysis can be used for making up a plan of the problem solution.

Direct analysis means that the problem is looked upon as consisting of a number of simpler problems the solution of which leads to the solution of the whole problem. In the course of direct analysis, the though moves from the given information to the question.

A series of questions for direct analysis of a problem may have the following sequence:

– Read the question of the problem. Can we answer the problem question right away?

– Characterize the composition of the problem. What kind of problem is it?

– Single out the first simple problem.

– What operation should be used to solve it?

– Why?

– Write down the solution of the first simple problem.

– Have we answered the question of the original problem?

– Single out the second simple problem.

– What operation should be used to solve it?

– Why?

– Write down the solution of the second simple problem.

– Tell us the answer to the second simple problem.

– Have we answered the question of the original problem?

– Tell me what plan is used to solve the original problem.

In backward analysis the thought moves from the question to the given information; each time it is necessary to specify what values are to be known in order to calculate the unknown value. Backward analysis might be supported by the following scheme:

– Read the question of the problem.

– Can we answer the problem question right away?

– What two values are to be

known to answer the question of the problem?

– What value is already known?

– What is unknown?

– What values are to be known to calculate how many pears there were in the vase?

– Which of these values are already known?

– Tell us the plan of the problem solution.

Mixed analysis partially employs both its kinds. It has no fixed algorithm:

– Can we answer the problem question right away?

– Why?

– Can we find out how many pears there were in the vase?

– Why?

– Tell us the plan of the problem solution.

Each problem analysis is accompanied by making up a plan for the problem solution followed by writing down the solution itself which is, actually, the next stage of work over a mathematical problem.

Let us see what forms of writing down the solution of a problem are possible.

The problem may be solved orally; in this case the pupil tells the teacher about all actions leading to the solution. But much more often the solution is written down. In doing this, it is common practice to write down the solution by operations. Different kinds of writing down the solution are distinguished: without commentary, with a brief commentary and with a detailed commentary. It is possible to write down the solution of a problem in the form of a plan. This kind of writing down the solution presupposes putting down sentences showing what values and in what order should be found in order to solve the problem.

The solution of a problem may be recorded with the help of a computer program.

The solution of a geometrical problem should be accompanied by constructing the required figure with the help of instruments.

Checking the problem solution is an important stage and a generalized skill of problem solving. This kind of work is not often used in school practice. But in view of formation of the general technique of problem solving and such important universal learning action as control this stage of work over a mathematical problem should be carried out much more often and specifically taught by acquainting children with different kinds of checking the problem solution. For example, it is possible to use the following technique. The teacher may give the problem situation and its solution (the method of writing down the solution may be varied). The children may be asked to check the solution by one of the known methods. This technique will not take much time and will teach the children to carry out checking regularly.

Let us consider the methods of checking the problem solution which should be acquired by schoolchildren.

Four main methods of checking the problem solution are described in

the methods literature: making up a problem reverse to the given one and solving it; parallel solution of a problem using a different (alternative) method; checking the problem solution according to all conditions and weighing up the result.

After checking the problem solution it is advisable to pay attention to some aspects that make this stage significant and accessible. It is necessary to teach children to make a conclusion about the correctness of the original problem solution after checking it. For example, after having solved a problem using a different solution path it is useful to make a conclusion with the help of such a sentence: "Solving the problem with the help of a different method we got the same answer we received using the first method; hence, the original problem has been solved correctly". The second moment consists in the fact that we should foresee the difficulty of the problem which is made up in order to check the original one. It must not be more difficult than the original one. Otherwise this method of checking will not be used by children in their independent work.

Let us consider the algorithms of schoolchildren's activity while using different kinds of checking the problem solution in more detail.

Making up a reverse problem and solving it. Having solved the original problem the pupil makes up a reverse problem, and while doing so performs the following actions.

1. Puts down the found number (answer to the original problem) in the text of the original problem.
2. Chooses a number from the given numbers of the original problem and makes it an unknown value in the problem reverse to the original one.
3. Formulates the text of the reverse problem.
4. Solves it.
5. Compares the answer to the reverse problem with the number which was chosen as the sought-for one in the original problem. If these numbers coincide the pupil makes a conclusion that the original problem was solved correctly.

This method of checking the problem solution can be used when the reverse problem is at least not more difficult than the original one and its solution does not cause any difficulties, i.e. the skill or technique of solution of problems of a certain kind has been formed well enough. But the analysis of practical experience shows that children do not use this method of checking the problem solution in their independent work because it is more difficult for them than the solution of the original problem. Consequently, this method of checking cannot be used for formation of self-control. It is more logical to carry out this method of checking under the supervision of a teacher because it is often necessary to check the text and the solution of the reverse problem.

Solving the problem with the help of a different method is the most widely used method of checking the problem solution. The essence of this method consists in looking for an alternative path to solve the problem after having solved it. After solving

the problem with the help of a different method the pupil compares the obtained result with the answer to the original problem and makes a conclusion about the correctness of the original problem solution. In order not to make the solution of the problem with the help of a different method appear as a means of evaluation it is necessary that the alternative method be easier and known by the pupils better than the first method.

As our observation shows, this method of checking is used by the children of primary school only under the guidance of the teacher.

Checking the problem solution according to all conditions or correlating the obtained result with the problem situation. The essence of checking consists in formulation of conclusions about the text of the problem accompanied by arithmetic calculations or without them. This kind of checking has a non-formal character, and the reasoning is based on understanding by the pupil who carries out checking of all words and relations given in the text of the problem. There is no clear algorithm of conducting such checking. Each problem needs a special line of reasoning.

Weighing up the results is one more method of checking the problem solution. Its essence lies in the fact that prior to the problem solution the pupils give an approximate result on the basis of preliminary analysis of the text. During the search of solution and its completion, the pupils have a chance to correlate the answer to the problem with the previously predicted result. The more exact the prediction, the higher its predictive functions. This method does not check the exactness of the answer. Its merit consists in the fact that it does not make the pupils check only the answer but also the process of the problem solution, which means that it does not only form the skills of final control but also those of current control. This kind of checking the problem solution is suitable for children in their self-control activity.

The technique of building various problem models after the solution of the problem may be considered to be the fourth method of checking the problem solution.

We have described the stages of work over a mathematical problem, which reflect the whole cycle of learning activity. We showed that the above mentioned stages are identical to the components of the general method of solving problems. We believe that teaching the general method of solving problems should not be concentrated on a simultaneous realization of each stage of problem solution and getting the result at a lesson, but on a step-by-step formation of each skill of the general method of solving problems in various conditions.

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